Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics
Standard level
Paper 1

Tuesday 10 May 2016 (afternoon)

Candidate session number

1 hour 30 minutes

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- You are not permitted access to any calculator for this paper.
- Section A: answer all questions in the boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics SL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[90 marks]**.

2216 – 7305

15 pages

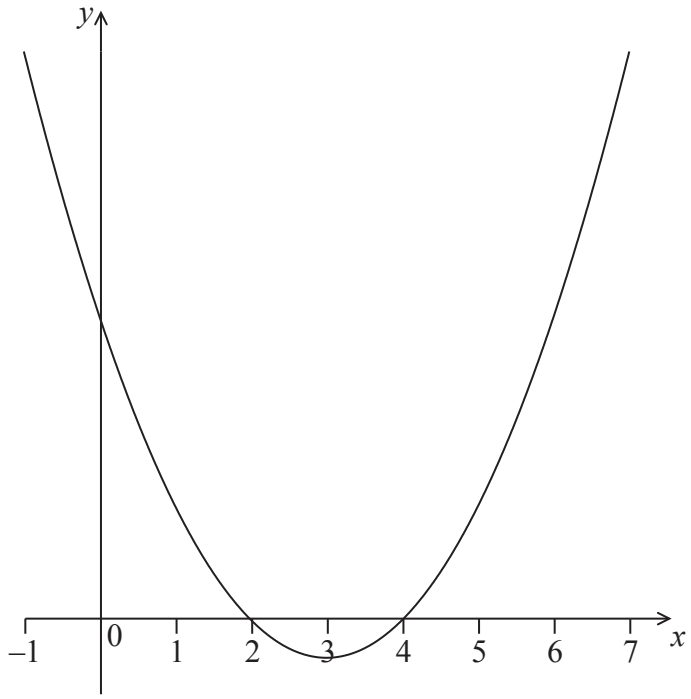
Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A

Answer **all** questions in the boxes provided. Working may be continued below the lines if necessary.

1. [Maximum mark: 6]

The following diagram shows part of the graph of a quadratic function f.



The vertex is at $(3, -1)$ and the x-intercepts at 2 and 4.

The function f can be written in the form $f(x) = (x - h)^2 + k$.

(a) Write down the value of h and of k. [2]

The function can also be written in the form $f(x) = (x - a)(x - b)$.

(b) Write down the value of a and of b. [2]

(c) Find the y-intercept. [2]

(This question continues on the following page)

(Question 1 continued)

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2. [Maximum mark: 5]

There are 10 items in a data set. The sum of the items is 60.

(a) Find the mean. [2]

The variance of this data set is 3. Each value in the set is multiplied by 4.

(b) (i) Write down the value of the new mean.

(ii) Find the value of the new variance. [3]

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3. [Maximum mark: 6]

Let $x = \ln 3$ and $y = \ln 5$. Write the following expressions in terms of x and y.

(a) $\ln\left(\frac{5}{3}\right)$. [2]

(b) $\ln 45$. [4]

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4. [Maximum mark: 6]

Three consecutive terms of a geometric sequence are $x - 3$, 6 and $x + 2$.
Find the possible values of x.

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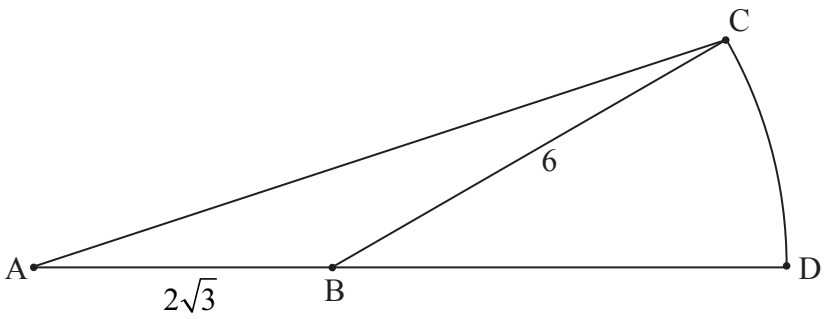
5. [Maximum mark: 8]

The following diagram shows a triangle ABC and a sector BDC of a circle with centre B and radius $6\,\text{cm}$. The points A, B and D are on the same line.

diagram not to scale



$AB = 2\sqrt{3}\,\text{cm}$, $BC = 6\,\text{cm}$, area of triangle $ABC = 3\sqrt{3}\,\text{cm}^2$, $A\hat{B}C$ is obtuse.

(a) Find $A\hat{B}C$. [5]

(b) Find the exact area of the sector BDC. [3]

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6. [Maximum mark: 7]

Let $f(x) = 6x\sqrt{1-x^2}$, for $-1 \leq x \leq 1$, and $g(x) = \cos(x)$, for $0 \leq x \leq \pi$.
Let $h(x) = (f \circ g)(x)$.

(a) Write $h(x)$ in the form $a\sin(bx)$, where $a, b \in \mathbb{Z}$. [5]

(b) Hence find the range of h. [2]

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7. [Maximum mark: 7]

Let $\boldsymbol{u} = -3\boldsymbol{i} + \boldsymbol{j} + \boldsymbol{k}$ and $\boldsymbol{v} = m\boldsymbol{j} + n\boldsymbol{k}$, where $m, n \in \mathbb{R}$. Given that $\boldsymbol{v}$ is a unit vector perpendicular to $\boldsymbol{u}$, find the possible values of m and of n.

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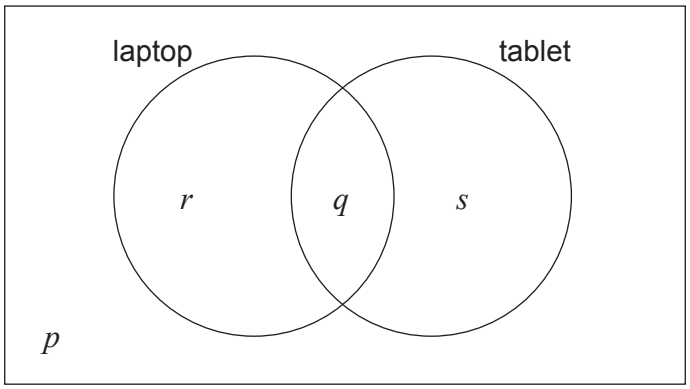
Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

8. [Maximum mark: 13]

In a class of 21 students, 12 own a laptop, 10 own a tablet, and 3 own neither.
The following Venn diagram shows the events "own a laptop" and "own a tablet".
The values p, q, r and s represent numbers of students.



(a) (i) Write down the value of p.

(ii) Find the value of q.

(iii) Write down the value of r and of s. [5]

(b) A student is selected at random from the class.

(i) Write down the probability that this student owns a laptop.

(ii) Find the probability that this student owns a laptop or a tablet but not both. [4]

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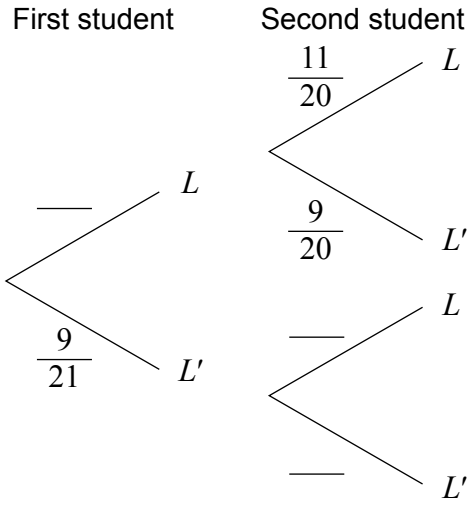
(Question 8 continued)

(c) Two students are randomly selected from the class. Let L be the event a "student owns a laptop".

(i) **Copy** and complete the following tree diagram. (Do **not** write on this page.)



(ii) Write down the probability that the second student owns a laptop given that the first owns a laptop. [4]

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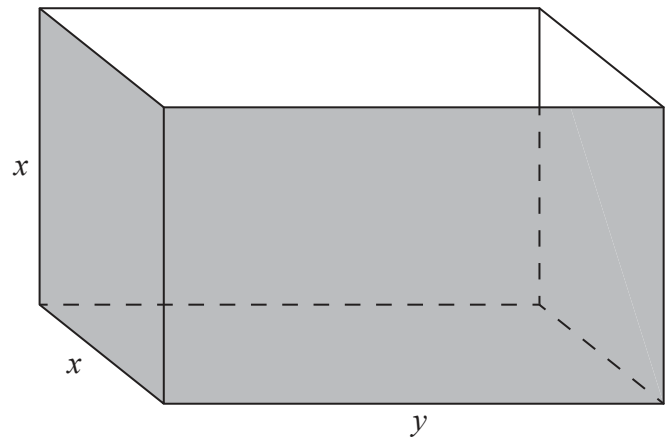
9. [Maximum mark: 16]

Fred makes an open metal container in the shape of a cuboid, as shown in the following diagram.



The container has height x m, width x m and length y m. The volume is $36\,\text{m}^3$.

Let $A(x)$ be the outside surface area of the container.

(a) Show that $A(x) = \frac{108}{x} + 2x^2$. [4]

(b) Find $A'(x)$. [2]

(c) Given that the outside surface area is a minimum, find the height of the container. [5]

(d) Fred paints the outside of the container. A tin of paint covers a surface area of $10\,\text{m}^2$ and costs $\$20$. Find the total cost of the tins needed to paint the container. [5]

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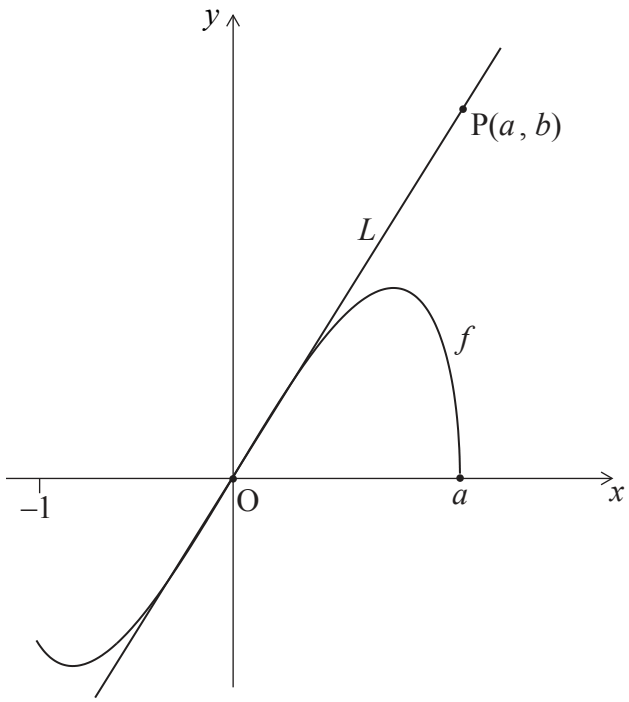
10. [Maximum mark: 16]

The following diagram shows the graph of $f(x) = 2x\sqrt{a^2 - x^2}$, for $-1 \leq x \leq a$, where $a > 1$.



The line L is the tangent to the graph of f at the origin, O. The point $\mathrm{P}(a, b)$ lies on L.

(a) (i) Given that $f'(x) = \dfrac{2a^2 - 4x^2}{\sqrt{a^2 - x^2}}$, for $-1 \leq x < a$, find the equation of L.

(ii) Hence or otherwise, find an expression for b in terms of a. [6]

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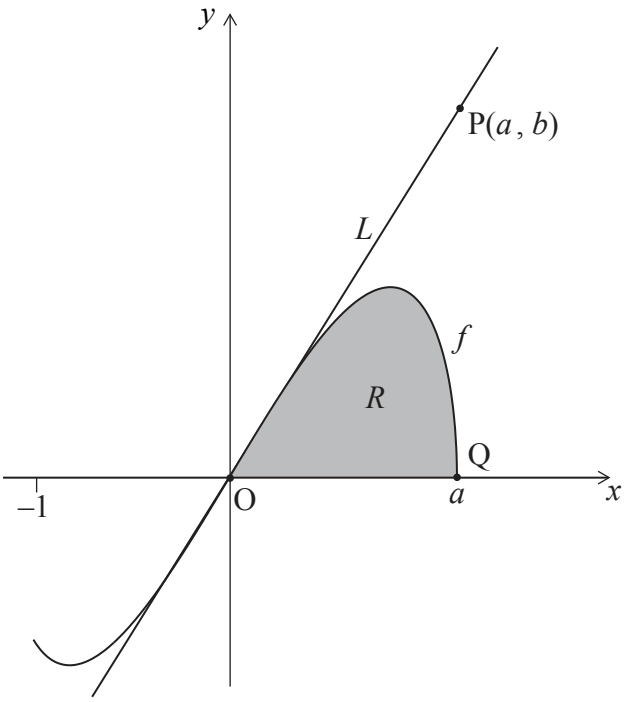
(Question 10 continued)

The point $\mathrm{Q}(a, 0)$ lies on the graph of f. Let R be the region enclosed by the graph of f and the x-axis. This information is shown in the following diagram.



Let A_R be the area of the region R.

(b) Show that $A_R = \frac{2}{3}a^3$. [6]

(c) Let A_T be the area of the triangle OPQ. Given that $A_T = kA_R$, find the value of k. [4]

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